Roll No. | | | | | | | | | | |

Total No. of Questions : 09 Total No. of Pages : 02

MCA (Sem.–1)

# DISCRETE STRUCTURES AND OPTIMIZATION

Subject Code : PGCA-1917

M.Code : 79035

Date of Examination : 14-12-2023

Time : 3 Hrs. Max. Marks : 70

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying TEN marks each.
4. Select atleast TWO questions from SECTION - B & C.

## SECTION-A

1. **Write short notes on the following :**

a) What is the basic concept of set theory and how it is used in discrete mathematics?

b) How does the study of combinatorial mathematics contribute in optimizing algorithms in computer science?

c) What distinguishes fields, integral domains and rings in abstract algebra?

d) What is the role of hashing functions in data structures?

e) What are some practical instances where the pigeonhole principle is employed within combinatorial mathematics?

f) Differentiate briefly between semigroups, monoids and groups.

g) What is the significance of cosets and congruence relations in the context of group theory?

h) Define isomorphism and homomorphism in graph theory.

i) List the characteristics of Hamiltonian paths.

j) What is the concept of Euler graphs and list their properties?



## SECTION-B

2. Imagine a scenario where the population of bacteria in a colony multiplies by 4 every hour.

a) How would you establish a recurrence relation for the number of bacteria after n hours?

b) If a colony begins with 50 bacteria and the population grows by a factor of 5 every hour, how many bacteria will be present in the colony after 8 hours?

3. Explain the practical applications of combinations and permutations. What is the minimum number of students needed in a discrete mathematics class to guarantee that at least three students receive the same grade, when there are four possible grades: A, B, C, and D?

4. Define Boolean Algebra, sub-algebras and Boolean rings. Given the Boolean function $F(A, B, C, D) = \sum(0, 2, 5, 7, 11, 13, 14, 15)$, employ the Karnaugh Map method to simplify the function F and represent the resulting expression.

5. Provide an overview of the key characteristics of rings and offer examples of different types of rings. Also, compare rings to other algebraic structures, such as groups and fields, highlighting their distinctive features.

## SECTION-C

6. Describe the fundamental principles and characteristics of right cosets in the context of group theory. Calculate the number of right cosets of a subgroup H in a group G, given $|G| = 72$ and $|H| = 8$.

7. Delve into the concept and attributes of symmetric groups and provide concrete examples. Determine the order of a symmetric group S12 and identify any of its non-trivial subgroups.

8. Investigate the significance and practical applications of graph isomorphism in graph theory. Provide specific instances where graph isomorphism plays a crucial role in real-world problems.

9. Write an extensive explanation of the concept of planar graphs and their coloring properties in graph theory. Discuss the history and significance of the Four-Color Theorem and the current status of its proof, whether it remains a conjecture or has been proven.

**NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.**

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**MCA (Sem.–1)**

# DISCRETE STRUCTURES & OPTIMIZATION

**Subject Code : PGCA-1917**

**M.Code : 79035**

**Date of Examination : 07-01-2025**

**Time : 3 Hrs.**

**Max. Marks : 70**

**INSTRUCTIONS TO CANDIDATES :**

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying TEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

## SECTION-A

**1. Write short notes on :**

a) Find generating function for series $-3, 9, -27 \ldots$

b) Give an example of relation which is neither reflexive, nor symmetric, nor transitive, nor anti-symmetric.

c) A graph has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices of graph G.

d) Find g of Where $f(x) = 2x + 1$ and $g(x) = x^2 - 2$.

e) Give an example of a relation which is neither reflexive. Prove that $F(x) = 3x + 5$ for all $x \in R$ is invertible, where $f \rightarrow R$ R.

f) Write Dual of each Boolean Equation (i) $(a + 1) \cdot (0 + a') = 0$ (ii) $a + a'b = a + b$.

g) Find the greatest lower bound and upper bound of the sets $\{3, 9, 12\}$ and $\{1, 2, 4, 5, 10\}$, if they exist in a POSET $(Z^+, |)$.

h) Define fields with example.

i) Define B-trees.

j) What do you mean by commutative ring?



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## SECTION-B

2. a) Let R be relation on the set of ordered pair of positive integers such that $(x, y)$ R $(u, v)$ if $xv = yu$. Show that R is an equivalence relation.

   b) Draw Hasse diagram representing the partial ordering

   $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 12, 24, 48, 72\}$

3. a) How many bit strings of length seven either begin with two 0s or end with three 1s?

   b) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

4. a) Define an ideal of a ring. Discuss ring homomorphisms and the connection with ideals.

   b) Use K-maps to minimize the sum of products expansion $xyz + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z$.

5. Solve Recurrence Relation $a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3$.

## SECTION-C

6. a) State and prove Euler's formula.

   b) Show that an undirected graph has an even number of vertices of odd degree.

7. a) State and prove Lagrange's theorem.

   b) Define cut points and spanning tree along with examples.

8. Prove that $(Z, *)$ is an abelian group.

9. a) Describe three different methods that can be used to represent a graph.

   b) Draw a simple, graph with at least five vertices and eight edges. Illustrate how it can be represented using the methods that are described in part (a).

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